True Approximations for k-Center with Covering Constraints

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Motivation

- Surge of interest in fairness-inspired k-center versions
- Fairness conditions naturally lead to covering constraints
- Current techniques only give pseudo-approximations

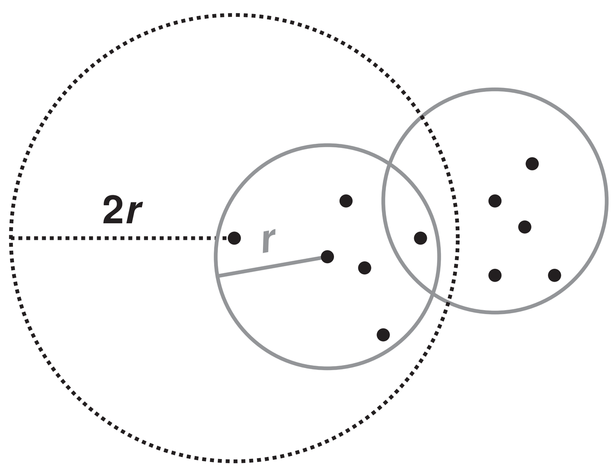
How to deal with covering constraints in k-center problems?

Recap: The k-Center problem (classical version)

Task: cover all points of a metric space with k balls of smallest possible radius

2-approximation can be achieved by:

- Pick arbitrary point
- Remove ball of radius $2r$
- Repeat



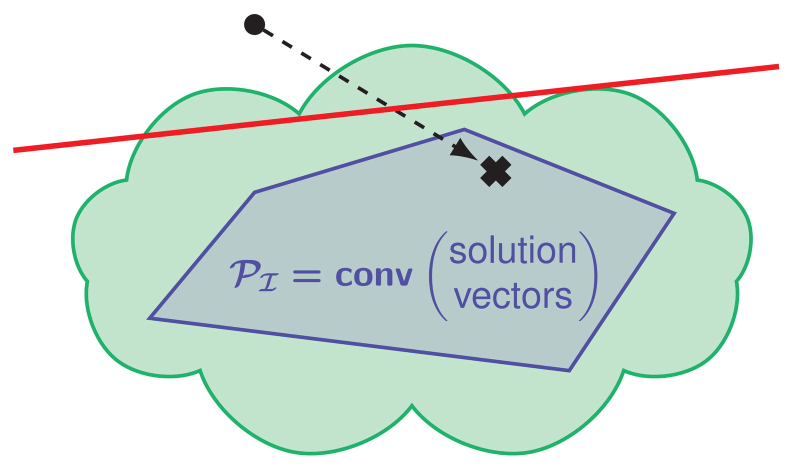
Recap: round-or-cut (classical framework we build upon)

Rounding technique based on Ellipsoid Algorithm:

Find such that we can

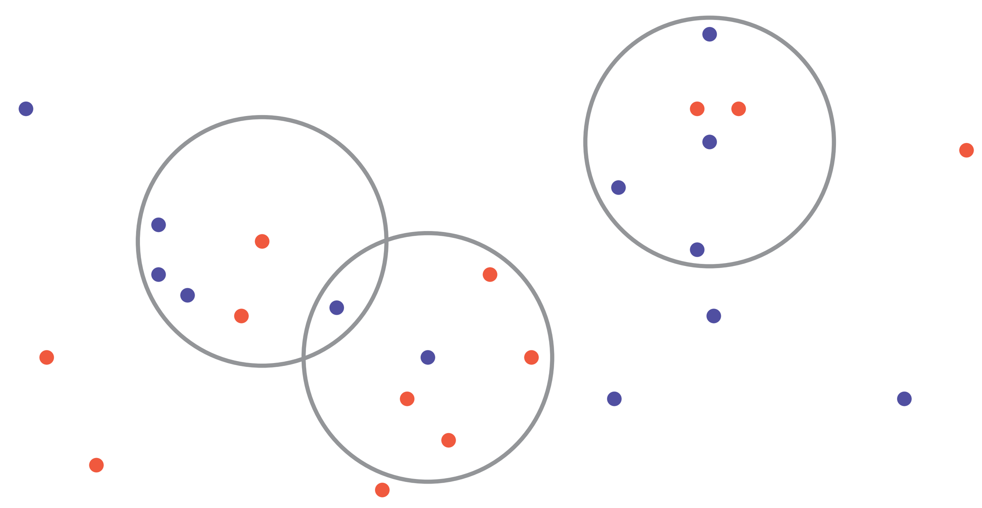
- either round solution (if in)
- or separate from target polytope

Then use this in Ellipsoid iterations



γ-Colorful k-Center Problem (γCkC), introduced by [1]: an illustrative example for our approach

- Input: metric space X, color classes $X_1, \dots, X_\gamma \subseteq X$ with covering requirements $m_1, \dots, m_\gamma$
- Output: centers $C \subseteq X$ with $|C| = k$ and $|\bigcup_{c \in C} B(c,r) \cap X_\ell| \geq m_\ell$
- Goal: minimize r



$$\mathcal{P} = \left\{ (\overset{\text{extent to which points are covered by centers}}{x}, \underset{\text{extent to which points are opened as centers}}{y}) \in [0,1]^X \times [0,1]^X \;\middle|\; \begin{array}{ll} \sum_{u \in X} y(u) \leq k & \\ \sum_{v \in B(u,r)} y(v) \geq x(u) & \forall u \in X \\ \sum_{u \in X_\ell} x(u) \geq m_\ell & \forall \ell \in [\gamma] \end{array} \right\}$$

Prior Work

- $(17+\epsilon)$-approximation for plane [1]
- 2-pseudo-approximation opening $k+\gamma-1$ centers [1]
- 2-pseudo-approximation for Fair γ**CkC** [2]
- Round-or-cut first used by [3] in this context

Our Results for (Fair) γCkC

- 4-approximation for Fair γ**CkC** for any metric for $\gamma = O(1)$ (Fair γ**CkC** is a probabilistic generalization of γ**CkC** by [2])
- γ**CkC** is inapproximable for unbounded γ if $P \neq NP$, and for $\gamma = \omega(\log|X|)$ under *ETH*

Algorithm: Illustration for γCkC for $\gamma = 2$

start

Fractional solution $(x,y) \in \mathcal{P}$

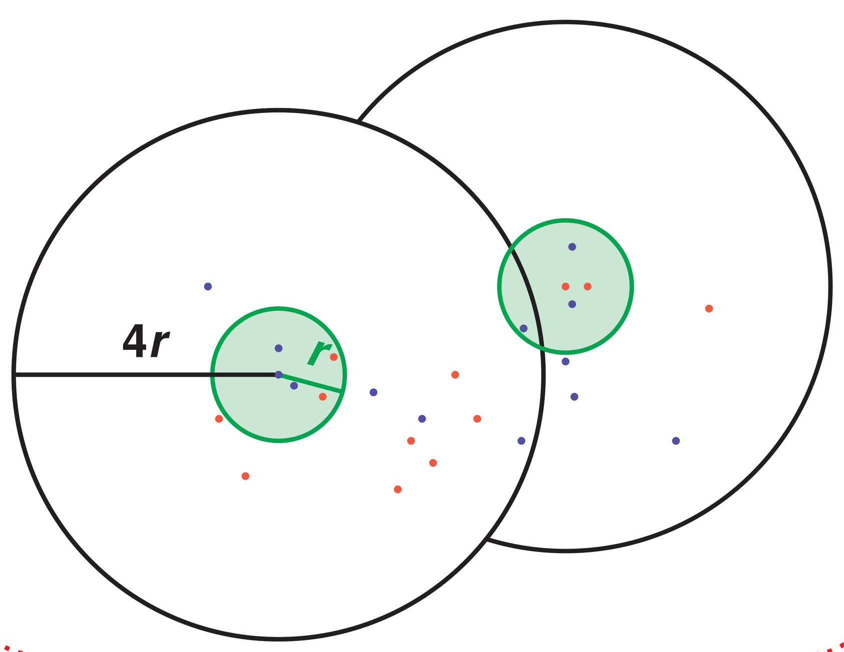
- In decreasing order of x
- Remove balls of radius $4r$
- = balls of radius r around cluster centers

y( ) small — YES: $4r$-solution; NO:

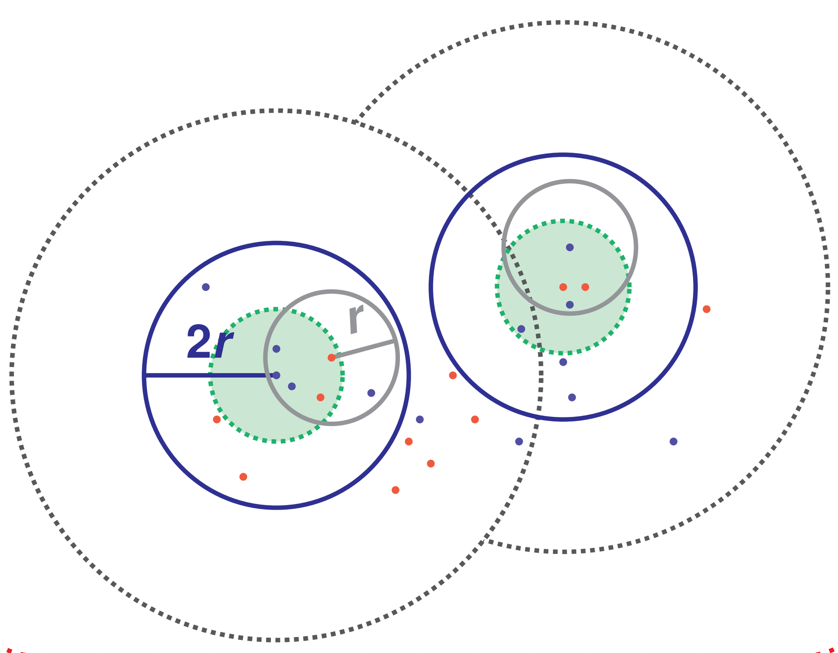
$\exists$ integral solution with y( ) large — YES: $2r$-solution; NO:

y( ) small separates (x,y) from $\mathcal{P}_\mathcal{I}$

- If y( ) $\leq k-1$, pseudo-approximation from [1] gives $4r$-solution



- Auxiliary problem: $\exists$ integral solution with all centers in ?
- If yes, then $\exists$ integral solution of radius $2r$ with centers in C
- Can find such a solution with a DP



Further Result

- 5-approximation for supplier version

Open Questions

- Best guarantee?
- Knapsack/Matroid γ**CkC**?

References

[1] S. Bandyapadhyay, T. Inamdar, S. Pai, and K. R. Varadarajan, "A constant approximation for Colorful k-Center", *ESA*, 2019.
[2] D. G. Harris, T. Pensyl, A. Srinivasan, and K. Trinh, "A lottery model for center-type problems with outliers", *ACM TALG*, 2019.
[3] D. Chakrabarty and M. Negahbani, "Generalized center problems with outliers", *ACM TALG*, 2019.